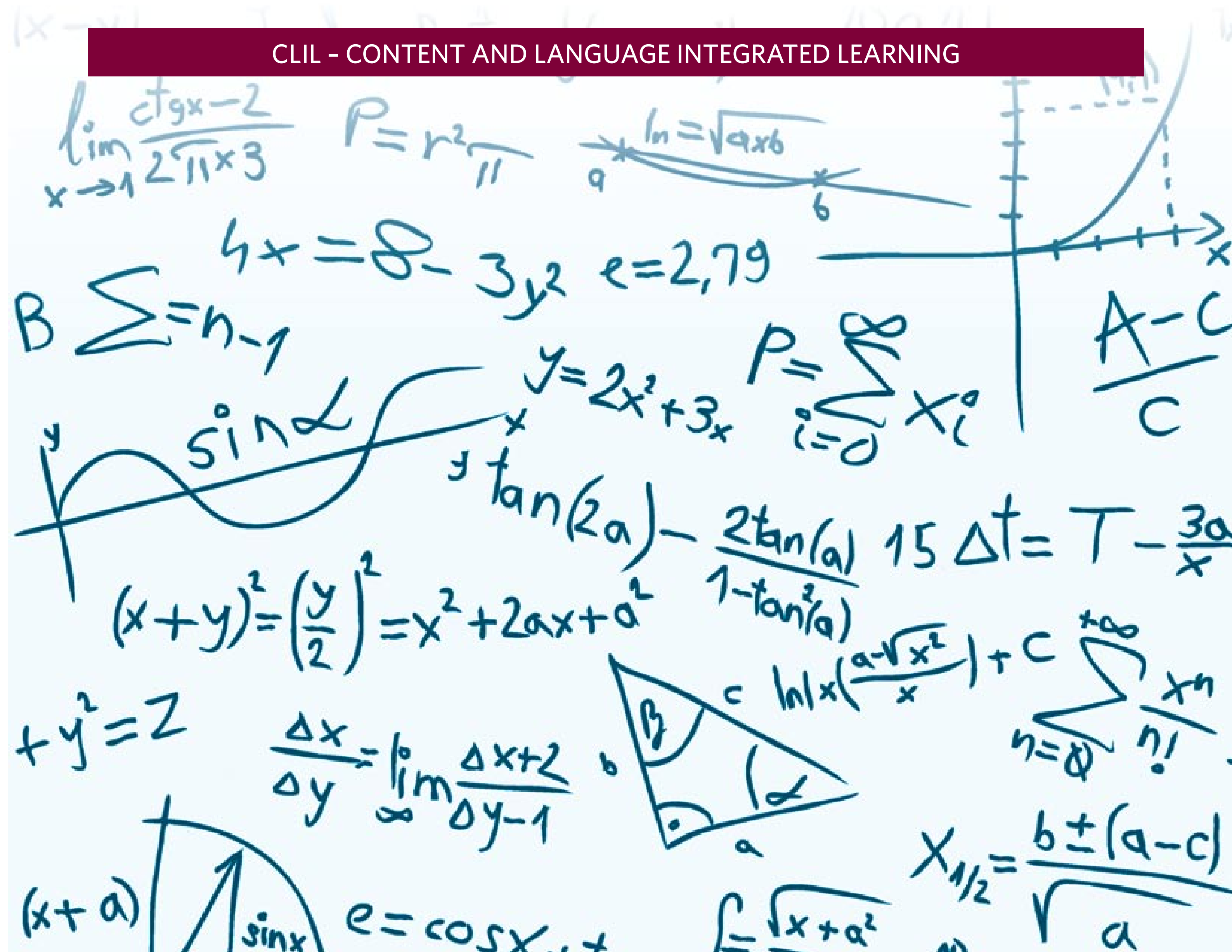
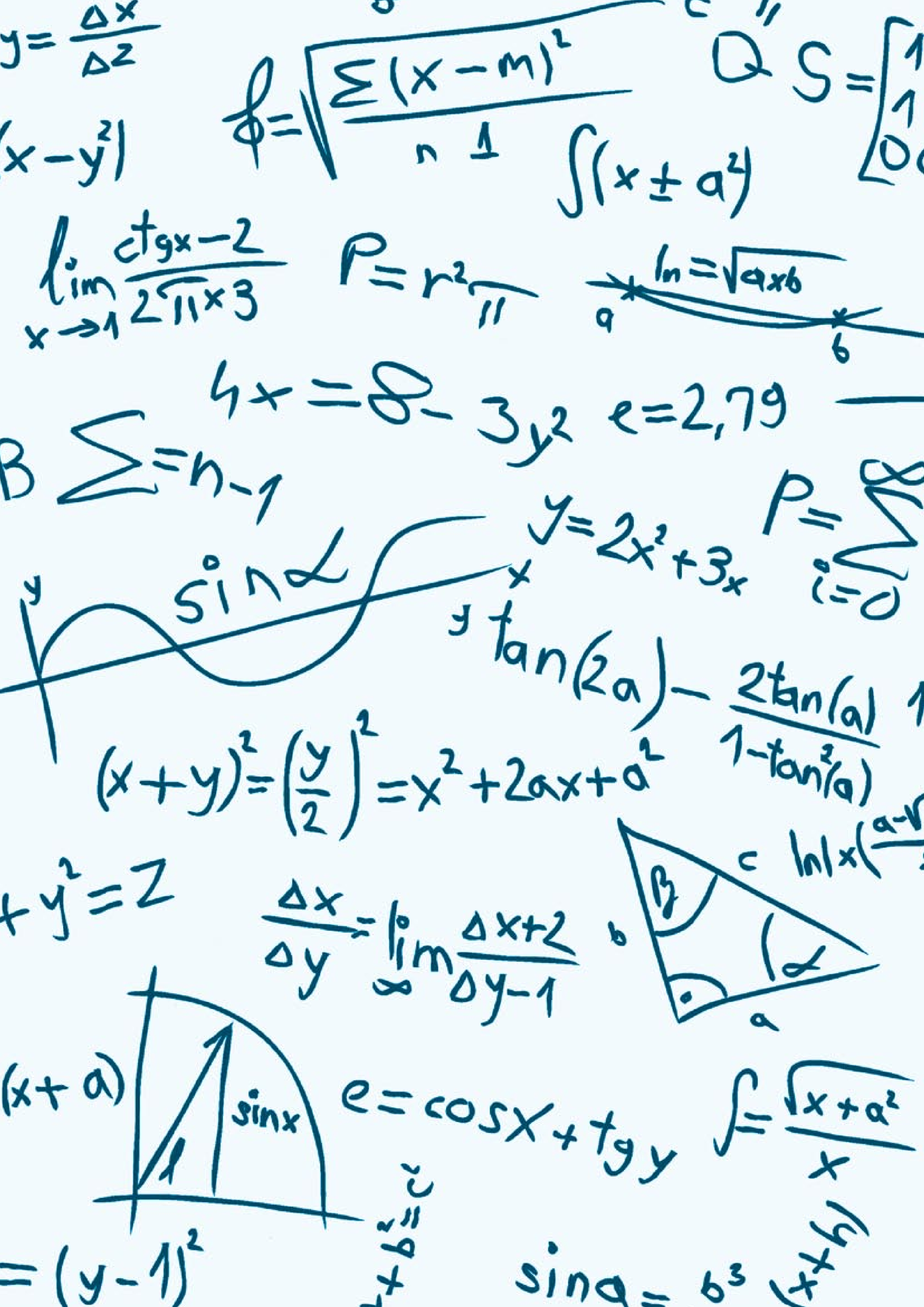
UNIVERSITY *of* CAMBRIDGE
ESOL Examinations

Cambridge English

Teaching Maths through English – *a CLIL approach*

CLIL – CONTENT AND LANGUAGE INTEGRATED LEARNING





y = Δx/Δz
Σ(x − m)²
n 1
∫(x ± a²)
lim ctgx − 2 / 2√11 × 3
x → 1
P = r²π
ln = √axb
4x = 8 − 3y²
e = 2,79
Σ = n − 1
sinα
y = 2x² + 3x
P =
i = 0
tan(2a) − 2tan(a) / 1 − tan²(a)
(x + y)² = (y/2)² = x² + 2ax + a²
y² = Z
Δx/Δy = lim Δx + 2 / Δy − 1
ln|x
(x + a)
sinx
e = cosx + tgy
√x + a² / x
(y − 1)²
sina = b³

Contents

What is CLIL?

CLIL is an acronym for content and language integrated learning. It consists of teaching a curricular subject through the medium of a language other than that which is normally used. In CLIL courses, learners gain knowledge of the curriculum subject while simultaneously learning and using the foreign language.

Content first

It is important to notice that 'content' is the first word in CLIL. This is because curricular content leads language learning. For example, learning about mathematics often involves learners in making a hypothesis and then proving whether this hypothesis is true or not. Maths teachers should be aware of the language the learners need to think through this process, make their hypothesis and then provide their proof. For example:

HYPOTHESIS	If a whole number ends in *0* or *5*, then we can divide it by *5* (it is divisible by *5*)
PROOF	*135* ends in *5*, which implies that we can divide it by *5* (which implies that it is divisible by *5*)

Teachers need to teach this language, or help learners to notice it, so that learners can then communicate it. Learners in CLIL often need to hear language models many times before they can produce language accurately.

The 4Cs of CLIL

It is helpful to think of Coyle's 4Cs of CLIL for planning lessons (Coyle, 1999).

① **Content:** What is the maths topic? e.g. algebra, ratio, linear graphs

② **Communication:** What maths language will learners communicate during the lesson? e.g. the language of comparison for comparing and contrasting graphs

③ **Cognition:** What thinking skills are demanded of learners? e.g. identifying, classifying, reasoning, generalising

④ **Culture** (sometimes the 4th C is referred to as **Community** or **Citizenship**):
Is there a cultural focus in the lesson, e.g. do learners from different language backgrounds calculate in the same way? What symbols do they use?
In multilingual contexts, it is important to take time to talk about methods used in different cultures represented by learners in the classroom.

Content-obligatory or content-compatible language?

Learners need to produce both content-obligatory and content-compatible language.

Content-obligatory language

Every subject has its own content-obligatory language. This is the subject-specific vocabulary, grammatical structures and functional expressions learners need to:

- learn about a curricular subject
- communicate subject knowledge
- take part in interactive classroom tasks.

Content-compatible language

This is the non-subject-specific language which learners may have already learned in their English classes and which they can then use in CLIL classes to communicate more fully in the subject.

For example, maths teachers could identify the following language for learning about linear graphs:

Content-obligatory language	Content-compatible language
linear graph, non-linear graph straight-line graph, curved graph x-axis, x coordinate y-axis, y coordinate the x and y ax<u>e</u>s I'll plot the coordinates on the graph.	the same, different line, point numbers letters of the alphabet (explaining) This means…

Teachers do not need to use the technical descriptions of this language. Usually content-obligatory language is described as subject-specific or specialist language.

Considerations when planning a CLIL lesson

Activating prior knowledge

It is helpful to start a lesson by finding out what learners already know about the curricular topic. Learners may know many facts about the topic in their L1 (first language) but may have difficulty explaining this knowledge in a second or third language. When brainstorming ideas about a new topic, expect learners to use some L1 and then translate.

Input and output

Teachers need to plan the input, i.e. the information that is being presented in the CLIL class. Will it be delivered orally, in writing, on paper, electronically? Is it for whole class work, group or pair work? Will it include practical demonstrations? Teachers also need to plan for learner output. How are learners going to produce and communicate the content and language of the lesson? Will it be communicated orally, in writing or using practical skills? Will it be done individually, in pairs or in groups? What will success for the learners look like?

Wait time

Wait time refers to the time teachers wait between asking questions and learners answering them. When subjects are taught in a non-native language, a longer wait time than usual may be needed, especially at the start of the course, and it is important that teachers allow for this so that all students can take part in classroom interaction.

Interactive pair or group work tasks

These include tasks that involve learners in producing key subject-specific vocabulary and structures in meaningful learning. This may be at word level, e.g. a pair work information gap or labelling activity, or at sentence level, e.g. pair work asking and answering activities. Groups of learners can give short presentations on different parts of the curricular topic either digitally or face-to-face. Activities should support processing of new content and language.

Cognitive challenge

Learners usually need considerable support to develop their thinking skills in a second language. They need to communicate not only the everyday functional language practised in many English classes, but they also need to communicate the cognitive, academic language of school subjects. In CLIL, learners meet cognitively challenging materials from the beginning of their courses.

Providing scaffolding, i.e. content and language support strategies which are appropriate but temporary, is therefore very important. For example, teachers can write sentence starters on the board to support skills of reasoning:

We found that the	graph equation	is ______ because _____.

E.G. We found the graph is linear because the coordinates make a straight line.
We found the equation $y = x^2$ is non-linear because the coordinates make a curved graph.

Providing effective scaffolding is a challenge to all CLIL teachers because learners vary in the amount of support they need and in the length of time the support is needed. In one subject some learners might need more support and for longer than in another subject.

Developing thinking skills

Teachers need to ask questions which encourage lower order thinking skills (LOTS), e.g. the what, when, where and which questions. However, they also need to ask questions which demand higher order thinking skills (HOTS). These involve the why and how questions and therefore require the use of more complex language. In CLIL contexts, learners often have to answer higher order thinking questions at an early stage of learning curricular content.

What kind of challenges are there in CLIL?

Challenges for teachers

Subject teachers need to feel confident about their English language level, especially if they have not used English for some time. For example in maths, subject teachers need to:

- be able to present and explain concepts in their subject area clearly and accurately
- check pronunciation of subject-specific vocabulary which may look similar to other words in English but have different pronunciation
- be able to use appropriate classroom language to question, paraphrase, clarify, encourage and manage their classes in English.

Language teachers may either decide to teach subjects in CLIL or be asked to. They need to feel confident about subject knowledge and subject skills related to that subject. For example in maths, language teachers need to:

- know how to explain mathematical ideas and procedures in ways that will deepen learners' understanding
- be prepared to answer learners' questions about subject material which may be unfamiliar to them
- widen their knowledge of maths vocabulary and its pronunciation.

Challenges for learners

Most learners need considerable support in the first two years of CLIL courses. Most teachers do not know how long learners will take to do tasks, complete worksheets or understand instructions and explanations until they have used materials for the first year. Learners are all different; some need support in order to understand subject concepts, while some need more support to communicate ideas about subject concepts. Learners may need differentiation of:

- input
- task
- support

The table below gives an example of the way a classroom activity can be differentiated for less able learners who are finding out about positive and negative integers in maths.

Types of differentiation	Examples
input	work with numbers between *-10* and *10* only
task	complete *8* questions instead of *12*
support	provide number lines for those that need them -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Differentiation is also necessary for more able learners. Teachers need to plan extension activities to develop learner autonomy and learners' higher order thinking skills in the subject area. This is when Information and Communications Technology (ICT) can be very useful for online learning activities such as webquests and independent fact-finding.

Use of L1

In CLIL, it is recognised that use of L1 by learners, and sometimes by teachers, is a bilingual strategy which helps learners communicate fluently. Moving between L1 and the target language, either mid-sentence or between sentences, is quite common for learners in CLIL. It is considered to be more than simply translating. Classroom observations show that use of L1 and the target language happens between learners in the following interactions:

- clarifying teachers' instructions
- developing ideas for curricular content
- group negotiations
- encouraging peers
- off-task social comments

It is important that teachers avoid using L1 unless they are in a situation when it would benefit or reassure learners. Some schools have a policy where no L1 should be used. If L1 is used by teachers, they should be able to justify it.

Lack of materials

One of the most common concerns of CLIL teachers is that they can't find appropriate materials for their classes. Either they cannot find anything to complement the work done in the L1 curriculum, or adapting native speaker materials takes too much time. Increasingly, publishers are producing resources for specific countries. However, as teachers gain more experience with CLIL, they generally start to feel able to adapt native speaker materials from websites and from subject-specific course books.

Assessment

CLIL assessment leads to much discussion. Teachers are unsure whether they should assess content, language or both. Different regions, different schools and different teachers assess in a variety of ways. What is important is that there is formative as well as summative assessment in CLIL subjects and that there is consistency in how learners are assessed across subjects in each school. Learners, parents and other colleagues need to know what learners are being assessed on and how they are being assessed.

One effective type of formative assessment is performance assessment. It involves learners in demonstrating their knowledge of content and language. For example, they could:

- explain to others how they solved a set of equations
- describe different graphs and evaluate how well they plotted the algebraic data.

Teachers observe and assess learners' performance using specific criteria. Performance assessment can involve individuals, pairs or groups of learners. As CLIL promotes task-based learning, it is appropriate that learners have opportunities to be assessed by showing what they know and what they can do. Performance assessment can also be used to evaluate development of communicative and cognitive skills as well as attitude towards learning. For example, teachers can look for evidence of justifying opinions (communication), reasoning (cognitive skills) and co-operative turn-taking (attitude).

How can CLIL teachers overcome the challenges they face?

What can teachers do?

What subject teachers can do

- use an online dictionary with an audio function to hear maths vocabulary pronounced, e.g. Cambridge School Dictionary with CD-ROM
- use a grammar reference book in order to practise producing complex sentences such as conditional forms, e.g.
  - If you multiply x by 10, you'll see that __________.
  - If you multiplied x by 10, you'd see that __________.
  - If you had multiplied x by 10, you would have seen that __________.
- make sure learners know the functional language needed to talk about their subject area, e.g. explaining data on graphs, describing cause and effect
- share the planning of medium-term work, i.e. work which involves more than one lesson.

What language teachers can do

- read about the curricular subject, its concepts and the skills of the subject, either online or in English and L1 books
- highlight all the subject-specific vocabulary learners need and record it in topic areas, e.g. symmetry, 2-D shapes, 3-D shapes
- practise delivery of curricular materials and predict questions learners might ask about the topics presented.

What both subject and language teachers can do

- if possible, plan curricular topics together so that both benefit from each other's area of expertise.

How can teachers plan for CLIL?

There are more components in a CLIL lesson plan than in a subject or a language lesson plan.

Learning outcomes and objectives

Teachers first need to consider the learning outcomes of each lesson, each unit of work and each course. What will learners know and understand about maths? What will they be able to do at the end of the lesson, unit or course that they didn't know at the beginning? What skills will they master and what attitudes about co-operation will they develop? Learning outcomes are learner-centred as they focus on what the learners can achieve rather than on what the teacher is teaching.

For example, in maths:

Learners should know…	Learners should be able to…	Learners should be aware of…
that the length of the radius is half the length of the diameter; the formulae for the area and circumference of a circle	calculate the area and circumference of different circles; label different parts of circles	the application of the formulae in everyday life, e.g. calculating the distance a bicycle can travel

Subject content

What content will learners revisit and what content will be new?

Learners need to hear subject-specific language more than once, so revisiting a new concept deepens understanding. For example, the mean, median and mode are often confused because the words are similar. Teachers therefore need to present learners with different tasks which demand the same use of concepts to revisit learning. While planning, teachers should also note any **anticipated difficulties** learners may have with content and language learning.

Communication

As CLIL promotes interactive learning, teachers need to plan pair work or group work activities so that learners can communicate the language of the subject topic. Communicative activities should be integrated during the lesson, rather than left to the end of the class.

Communicative activities can be:

- **short**, e.g. tell learners they have 3 minutes to work with a partner to name the angles on the board then estimate the size of them
- **longer,** e.g. tell learners they have 10 minutes to work with a different partner to draw four angles each, label them, and then check each other's work using a protractor. Finally, learners tell their partners how accurate their drawings are.

Thinking and learning skills

The development of both thinking and learning skills needs to be planned.

Do learners move from lower order thinking skills to higher order thinking skills during the lesson? Subject teachers need to plan the types of questions they will ask to develop both types of thinking. The table below provides some examples.

Lower order thinking questions	Purpose	Higher order thinking questions	Purpose
Which angles are acute and which are obtuse? How many degrees are there in a right angle?	to check understanding to review learning	Which of these are possible and which are impossible? a) a triangle with two obtuse angles b) a quadrilateral with four acute angles	to develop reasoning and analytical skills

CLIL teachers need to plan how to support learners in developing learning skills such as observing details, taking notes, editing work, summarising and planning how to do problem-solving tasks.

Tasks

Teachers need to think about the kind of tasks learners will do during the lesson. It is important to plan a range of tasks which require different challenges, e.g. less demanding tasks include ordering data according to length or weight, comparing lengths or weights and so on. More demanding tasks include applying subject knowledge to everyday or hypothetical problems, e.g. is the mass of a litre of water the same on the Moon as it is on Earth?

Language support

All teachers need to plan to support:

① the language of input
② the language of output

Sometimes support for input and output can be the same. It is useful to think of support at word, sentence and text levels. In maths, text level is less common because most maths problems are explained in two or three sentences. The table below shows some examples for the topic of probability:

Word-level support	Sentence-level support
Word bank: probability impossible not very likely possible likely very likely certain equal chance/equally likely	**Substitution table:** The | probability | of ________ is ___________ . | chance | | likelihood | **Sentence starters:** It is likely that ___________ . The probability of it happening is ___________ .

Materials and resources

In all teaching, teachers need to find or create materials and evaluate them to make sure that the content and language are suitable for the learners at the stage they are at. In CLIL most subject materials need adapting because of the complexity of language used in the instructions or in the activities themselves. This can also be an issue when teachers recommend websites for learners to access. Web links need to be checked to ensure that the language is comprehensible.

Cross-curricular links

CLIL promotes links with other subjects in the curriculum so teachers should plan to include references to learning similar content in other subjects. For example, if learners are studying measurement in maths, it is useful to find out if they are studying measurement of temperatures in geography or measurement of mass in science. Teachers can then consolidate learning about measurement presented in other subject areas. In the linear graphs example on page 13, teachers can make a link to geography by reminding learners about the importance of accuracy when reading map coordinates.

Assessment

In CLIL plans, it is important to link the assessment to the learning outcomes of the lessons. Many European CLIL courses use 'Can Do' statements as these are clear for both teachers and learners. Assessment criteria are therefore transparent. For example:

Learning outcomes *Most learners should:*	Assessment *Most learners can:*
know: • there are four stages in handling data • there are differences between discrete data and continuous data be able to: • collect and organise data from different sources • represent discrete data in bar charts • represent data in a range of diagrams	• collect data • organise data • represent data in a range of diagrams • interpret data accurately • draw, label and interpret bar charts • draw, label and interpret line graphs • draw, label and interpret pie charts

Teachers should keep ongoing records of continuous, formative assessment done through learner observation in the classroom. It is not necessary to record information about each learner during each lesson, but over a period of four to six weeks evidence of learners' progress needs to be recorded. Here is part of a record for formative assessment in maths. Teachers record the date they observed learners accomplishing the various tasks:

name	can find perimeter of regular shapes	can find perimeter of irregular shapes	can find area of regular shapes	can find area of irregular shapes	can measure radius of a circle	can calculate circumference of a circle	can calculate area of a circle

What helps learners learn?

Two different surveys carried out with secondary CLIL learners produced interesting findings (Bentley and Philips, 2007). The first set of questionnaires was completed by 14–15 year-old Spanish learners who were studying science in English. It was their second year of learning science and fifth year of learning English. Here are a few learner responses to the question 'What helps you learn science in English?'

- *"More vocabulary and more diagrams on the worksheets"*
- *"Give us more explanations"*
- *"Use easy words for the explanations and vocabulary"*
- *"Work with games"*
- *"The complicated words in English with the Spanish words next to the English"*
- *"Put the hard vocabulary in a side of the page in Spanish. Put more pictures."*
- *"Add a list of vocabulary and illustrations"*
- *"Maybe put the most difficult science words with translation"*

It is clear that the quantity and complexity of new science vocabulary was causing problems. Highlighting key content vocabulary with explanations can be helpful *(see page 13)*.

The second set of questionnaires asked learners to tick a list of factors that help them learn school subjects in English. The learners were aged between 13 and 16 and were from different Spanish schools implementing CLIL courses.

- pictures: 38%
- word lists: 18%
- use of computers: 19%
- friends: 36%
- translations: 49%
- diagrams: 19%
- teacher explanations: 56%

The results of the survey show, firstly, how important it is that teachers explain their subject content effectively and, secondly, the importance of including interactive tasks so that learners can support one another as they learn.

CLIL teachers also report that, at the start of courses, learners need a lot of scaffolding and encouragement to help them learn. This can be in the form of clearly presented step-by-step instructions or explanations, constructive feedback and use of language frames. Learners respond positively to meaningful contexts which **personalise learning**. They also need regular consolidation of new content and language.

Appropriate task types

There is a range of task types which teachers can use in CLIL. The important point is that learners have a variety of tasks to stimulate output of content and language. Some tasks are more time-consuming to create and take more time to complete, so teachers need to be aware of this.

It is useful to keep a list of task types and tick off the ones that have been used over a school term or a year. Here are some examples of task types:

circle / underline	describe and guess	information transfer	odd one out
classify	domino games	jigsaw	PowerPoint presentations
compare and contrast	find the mistake	label match	true / false
complete the diagram	gap fill	multiple choice	word searches and web searches
crosswords	identification keys - e.g. a flow diagram with questions which help learners identify 3-D shapes	sequence	yes / no - e.g. an elimination game to guess the angle: Is it less than 180º? (yes) Is it a right angle? (no) Is it an acute angle? (yes)

Questions teachers should ask about the tasks they use are:

- Which tasks motivate the learners?
- Which tasks involve interaction?
- Which tasks develop thinking skills for the subject I teach?
- Which tasks need language support?

Applying CLIL to a maths lesson

Linear graphs

Learning outcomes for linear graphs

- to understand that equations explain the relationship between coordinate pairs
- to be able to differentiate linear from non-linear equations
- to be able to draw linear graphs from an equation or table of values

> ***Establishing learning outcomes***
>
> *The topic, linear graphs, is introduced with three learning objectives. Many teachers like to write these on the board so that learners are clear about what they should achieve by the end of the lesson.*

Vocabulary needed for linear graphs

Content-obligatory language

Key words which teachers and learners need in order to understand the topic are highlighted below in **bold font** with short meanings and explanations specific to maths, e.g. **substituting**. Diagrams are included to show two specific types of linear graph.

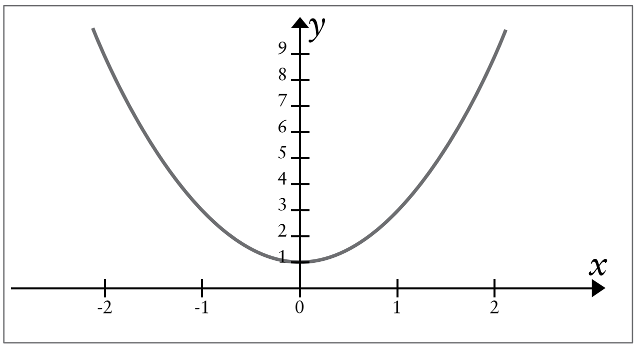
As a guide for teachers, there are two boxes with reasons for presenting vocabulary: the first to support maths vocabulary by inserting mathematical symbols; the second to clarify definitions through the use of diagrams.

- **BODMAS** is the order in which you do the operations:
  Brackets ( )
  Other (or **O**rder), e.g. square root √ or powers such as x^2 or x^3
  Divide ÷
  Multiply ×
  Add +
  Subtract –

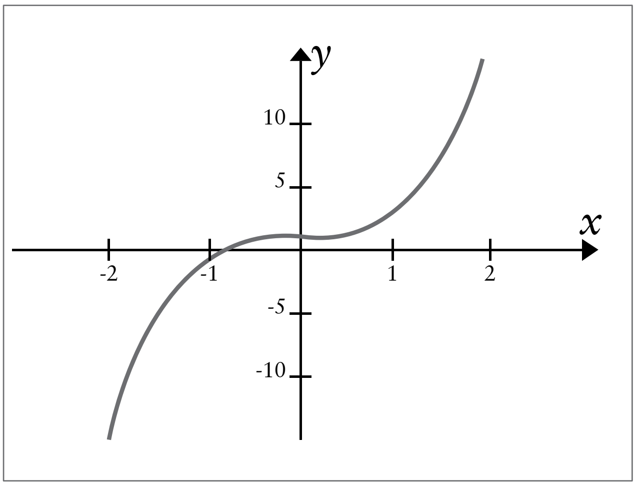
> ***Supporting input: use of mnemonics***
>
> *BODMAS is a 'word' or mnemonic which learners are likely to use to help them remember the order in which they should carry out calculations. Many mnemonics are used in maths.*

- **substituting** – replacing the letter in an equation with a number (following BODMAS rules)
- **straight-line graphs** are also called: **linear functions**, **linear graphs**, **linear equations** or **straight-line equations**
- **non-linear graphs** are not straight lines. They are **curves**, for example:

> *Including symbols such as:*
> ( ), √, x^2, x^3, ÷, ×, +, –
> *can help with understanding meanings.*

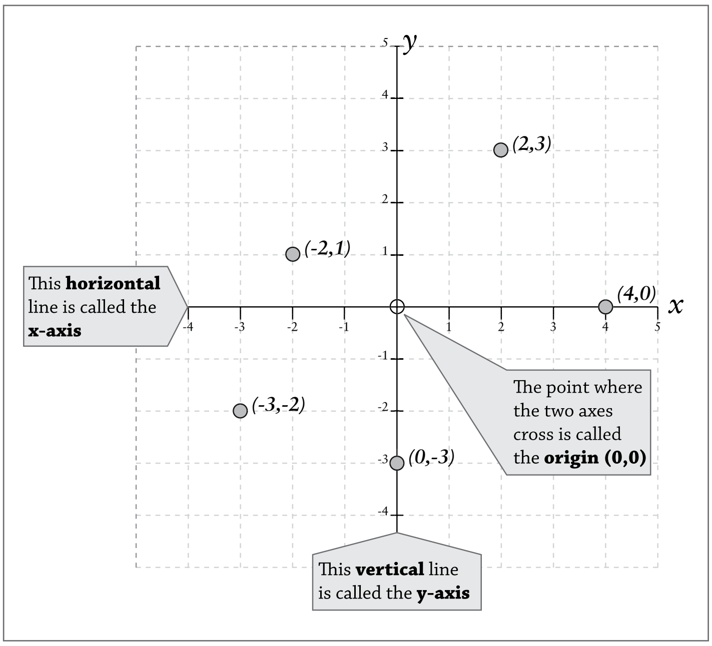


> *Diagrams can help clarify definitions.*



- **coordinates (x,y)** are also called **coordinate pairs** or **ordered pairs**. Coordinates are two numbers which describe the location of a point on a graph
- **x-axis** is the horizontal axis of a graph
- **y-axis** is the vertical axis of a graph
- **x-coordinate** is the first number in the pair. It tells you how far along the x-axis to move
- **y-coordinate** is the second number in the pair. It tells you how far up or down the y-axis to move
- **origin (0,0)** is the point where the two axes meet.
  The x-axis and the y-axis divide a plane into four quadrants
- **to plot** is to mark the position on a graph using the two coordinates, e.g. The coordinates are (3,4)
- the plural of axis is **axes**

Linear graphs coordinate diagram



Explanations of key subject concepts

*The coordinate diagram has explanations of the key concepts inserted around the quadrants. At the start of the CLIL course, it is better to keep explanations simple. This can be done by using active forms rather than passive, e.g. The **origin (0,0)** is the point where the two axes cross.*

Crossword puzzle

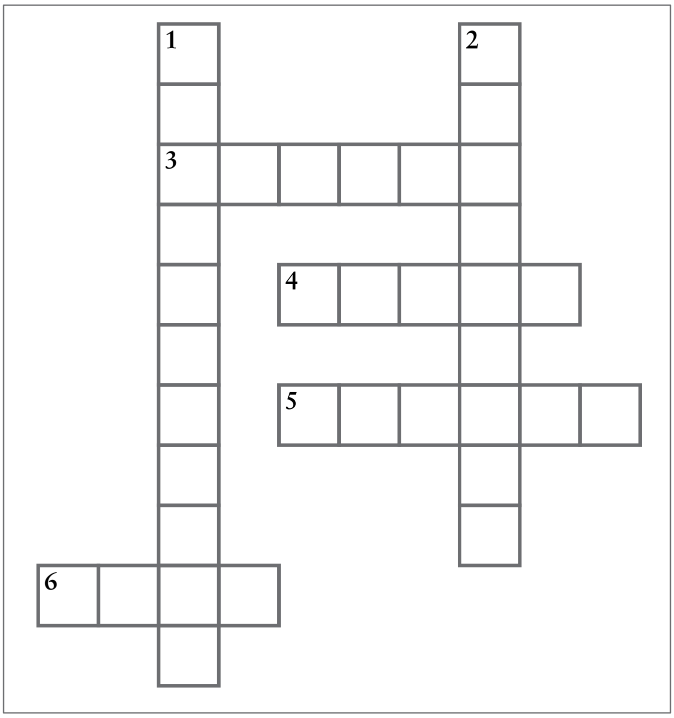
One way to consolidate learning of new maths vocabulary is a crossword puzzle. Explain that **across** means *write words in this* → *direction* and *down* means write *words in this* ↓ *direction*. Put learners into pairs, A and B. A has the clues across and B has the clues down. A asks B, 'What's 1 down?' B says 'points or ordered pairs of numbers'. B then asks A, 'What's 3 across?' A says 'the point (0,0)'. Continue until the puzzle is solved.

'Across' and 'down' are examples of **content-compatible** language which the learners will probably know from their English classes.

The maths vocabulary is **content-obligatory language** because learners must know these words to be able to understand the maths.

Revisiting content-obligatory language

There is a simple crossword with clues so learners can produce the vocabulary needed to study linear graphs. The crossword could be done as a pair work activity (student A has three clues in the crossword and student B has the other three clues. In turns they ask each other, e.g. What's 3 across?). This allows learners to speak as well as write the vocabulary.



Crossword clues:

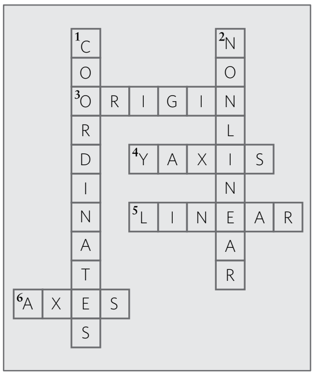
Across

3. The name of the point (0,0)
4. The name of the vertical line through the point (0,0)
5. The name for straight-line graphs
6. The plural of axis

Down

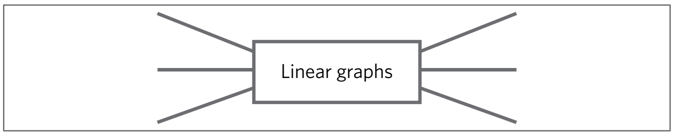
1. The name for a point or ordered pair on a graph
2. The name for a graph with a curve

Answer:



Activating prior knowledge for linear graphs

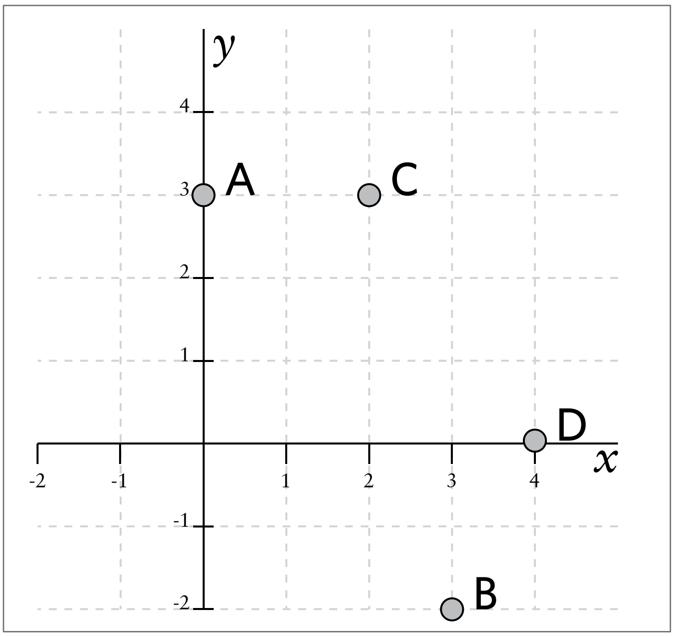
Brainstorm what learners already know about linear graphs. Draw a mind map on the board and ask them to give you facts they remember.



- using correct order of operations, i.e **BODMAS**-**B**rackets, **O**ther, **D**ivide, **M**ultiply, **A**dd, **S**ubtract)
  e.g. $2 + 3 \times 5 = 17$ (first 3×5, then $+ 2$)
- using negative numbers, e.g. $2 \times -3 = -6$ *or* $-4 \times -5 = 20$
- substituting into simple equations, e.g. $y = 4 - 2x$, find y when $x = 2$: $y = 4 - 2 \times 2 = 0$
- rearranging equations (changing the subject), e.g. make y the subject: $y + 3x = 4$, so $y = 4 - 3x$
- plotting coordinate pairs (x,y)

Evaluating prior knowledge task

① Calculate $10 - 4 \times 2$
② Find y when $x = 4$ in the following: $y = 4x - 3$
③ Find y when $x = -2$ in the following: $y = 8 - 3x$
④ Rearrange the following equations to make y the subject:
a) $y - 3x = 5$ b) $4x - 7 = 10y$
⑤ What are the coordinates of the points A, B, C and D in this diagram:



Activating prior knowledge

The five bullet points explain what learners should already know about linear graphs. The task evaluates prior knowledge but could also be a review of learning.

Before written evaluation takes place as in the example, it is helpful to start with an oral brainstorm involving the whole class so that learners have the opportunity to communicate content and language they already know. The simplest way is to draw a mind map on the board, ask learners for words about the topic and then record them.

⑥ Draw axes going from *-5* to *5* in the space below and plot the following points (labelling them H, I, J, K and L)
H = *(4,-1)* I = *(-1,-5)* J = *(-2,0)* K = *(5,4)* L = *(-3,2)*

What does a straight-line equation look like?

1. An enquiry approach (learner-led)

Ask learners to **compare** and **contrast** lists of linear and non-linear equations. This means they need to look for what is similar between all those in the first list and what is different between those in the second list. Ask questions such as 'What is the same about ...?' 'What is different...?'. You could use the following lists:

Linear equations

$y = 3x - 5,\ y = 0.4,\ y + x = \frac{1}{2},\ 3x - 2y = 4,\ y = x,\ x = -80$

Non-linear equations:

$y = x^2,\ y = 2x^3 + 4,\ y = \frac{1}{x},\ 3x^2 + y^2 = 55,\ y = \frac{3}{x},\ y = 3x + 2t$

Then you could give them two more equations that are different such as $x = 2$ and $x = y^2$, and ask learners to decide which lists these should be in.

Task types

There are examples of the following task types:

- *compare and contrast (what is the same and what is different)*
- *classifying (deciding which equations are linear and which are non-linear)*
- *listening and reading data (the teacher explains the differences between two sets of graphs while learners look at the different graphs).*

2. An investigative approach

Ask learners to research or look up the difference between linear and non-linear equations. You could direct them to the internet (see previous suggestions) or textbooks that explain it. You could then ask learners to give a presentation of their findings to the rest of the class. This encourages independent learning and study skills but if this is not an approach that your learners are used to you may need to give them help at first.

> ***Learning skills***
>
> *Learners are encouraged to find – either digitally or in books – select then present information about linear equations to the rest of the class. Clearly, support may be needed for this task.*

Alternatively, science experiments give learners opportunities to investigate whether or not a particular relationship produces a straight-line graph when plotted as coordinates, e.g. the relationship between electric current and voltage or the relationship between temperature and time when heating water.

3. A teacher-led approach

Give the learners the same two lists of graphs and explain the differences between the two sets of graphs:

- linear graphs can have only one letter, e.g. $y = 4$ or $x = -80$. Non-linear graphs always have more than one letter
- there are usually a few numbers in both sorts of graphs (the exception here is $y = x$)
- linear graphs have no more than two letters, e.g. $y = 3x - 5$ but NOT $y = 3x + 4t$
- in linear graphs there are no powers of x other than x^1, you don't see x^2 or x^3 or $\frac{1}{x}$ (which is x^{-1}).

> *The same applies for the y term in linear graphs, you don't see y^2 etc.*

Consolidation activity – card sorting

N.B. Card sorts can be adapted to suit the group or level of ability. In an easy card sort you can **classify** cards into two simple **categories** such as **linear** or **non–linear**. In a more difficult version for older learners, or to differentiate for more able learners, you could use the same cards but ask learners to classify the linear graphs into four further categories: **horizontal lines**, **vertical lines**, **lines with a positive gradient** and **lines with a negative gradient**. Note that they may need to investigate the meanings of these new terms first. You may find your more able learners have the ability and motivation to do this and would welcome more challenging work.

> ***Consolidating learning***
>
> *Card sorting is a very effective way of consolidating both new content and new learning. In this activity, learners classify information written on cards according to whether it describes a linear or non-linear equation and then into four given sub-categories.*

You can also differentiate by leaving some cards out for less able learners or by adding harder cards to challenge the more able learners. You could include a creative approach by asking learners to use creative thinking to add some further examples of their own. This is an example of an open-ended task. Learners can then be given the incentive to produce a difficult example by telling them their card will be given to another group for the other group to try to classify it.

> ***Differentiation***
>
> *Options for less able and more able learners are suggested: less able learners are given the meanings of the sub-divisions (horizontal lines, vertical lines, lines with positive and negative gradients); more able learners can find out the meanings of these terms.*

Produce cards, like these, on a word processor or spreadsheet. Print them then cut them out.

$y = 2 - 4x$	$y = -3$
$x^2 + y^2 = 8$	$y = \frac{3x}{5}$
$x = \frac{1}{y}$	$t = 3p$
$y = 3x - 6$	$y = 2$
$y = 0.6$	$x = -1$
$x = \frac{1}{3}$	$x = 0$

Don't forget some blank cards for learners to write their own equation for each of the types.

Produce a classifying sheet like this to sort the cards into groups.

Linear	*Non–linear*
Don't know	

If you include a 'don't know' box you can talk to learners about the cards they have put in the 'don't know' box.

Some suggested equations that you could make, classified into types, are:

Linear			Non-linear
Horizontal lines	Positive gradient	Negative gradient	
$y = -3$	$y = \frac{3x}{5}$	$y = 2 - 4x$	$x^2 + y^2 = 8$
$y = 0.6$	$t = 3p$	$y - 1 = -x$	$x = \frac{1}{y}$
$y = 2$	$y = 3x - 6$	$y = \frac{-x}{5}$	$y = x^2 + 8$
$0 = y$	$y - 9 = 7x$	$5y - 8 = -3x$	$y = x^2 + 3x - 7$
Vertical lines	$y + 3 = 4x$	$2y - 7 + 4x = 0$	$x^2 = y - 1$
	$y = x$	$y + 3x = 9$	$y = x^3 + 2x^2 - x$
	$2y - 4x = 7$	$3x = 5 - y$	$y = \frac{2}{x}$
$x = -1$	$x = 2 + y$	$x + y = 0$	
$x = {}^1/_3$	$m = 7 + 3p$		
$x = 0$	$c = 8p + 2$		
$7 = x$	$r = t$		

Learning skills

Two ideas have been put forward to develop learner autonomy: blank cards are included so learners can write their own equations; a 'don't know' classification is included so learners can record anything they are uncertain of.

How to draw linear graphs from an equation

There are four steps:

① Look at the equation – it tells you the relationship between the coordinates. For example:

$y = 2x$	This means to find the y-coordinate is double the x-coordinate or **x-coordinate multiplied by 2**
$x + y = 4$	This means the **x-coordinate and the y-coordinate add up to 4**
$x = 5$	This means the **x-coordinate is always 5**. The y-coordinate can be any number

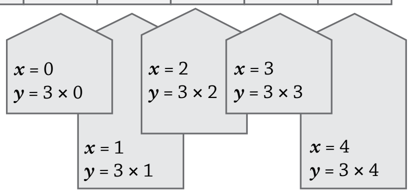
② Use these ideas to help find the coordinates

Using the equation $y = 3x$, what will the y-coordinate be when $x = 2$?	English to use *(keep explanations short, key phrases are in bold – the rest is extra English)*
$y = 3x$	**Use substitution**
$y = 3 \times 2$	**Replace the x with 2 and multiply**, as $3x$ means 3 multiplied by x
$y = 6$	So **when $x = 2$**, the **coordinate pair is $y = 6$** or the **point (2,6)**

③ Draw a table of values
Completing the **table of values** we get:

x	0	1	2	3	4
y	0	3	6	9	12

$y = 3x$



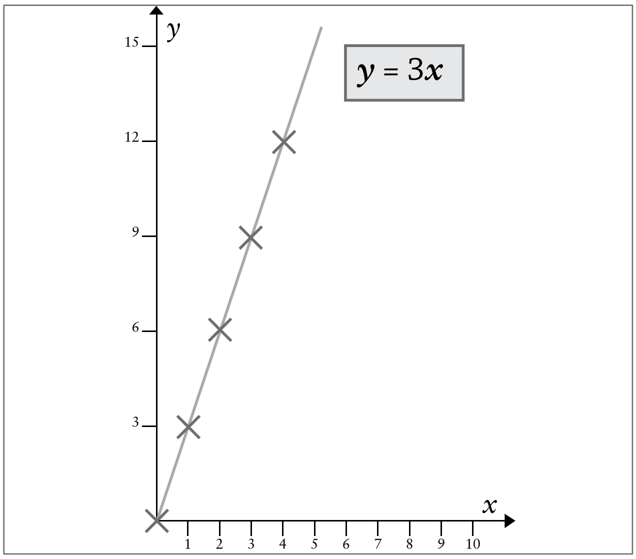
④ Turn the table of values into coordinates to plot and draw a straight line

x	3
y	9

This means you need to plot the point (3,9)

> ### *Scaffolding*
> *This section includes strategies to scaffold learning. These include breaking tasks down into clearly numbered steps (1- 4), highlighting key vocabulary and phrases the teacher can use to present the language of maths and labelled tables with explanations.*

Now plot the points and draw a STRAIGHT line with a ruler:



Real–life examples

Ⓐ '*PURPLE*' is a mobile phone company. It charges: $\$5$ each month for rental and $\$0.10$ each minute. The cost of the calls each month is c. The number of minutes each month is m. The equation for the cost of the calls is:

$c = 0.1m + 5$

Draw the vertical axis from 0 to $\$20$ to show c. Draw the horizontal axis from 0 to 120 minutes to show m. Now draw the graph of $c = 0.1m + 5$

Ⓑ A different mobile phone company, '*BLUE*', offers free calls but their monthly rental is $\$15$ per month. Draw the graph of this on the same axes as in part Ⓐ

Ⓒ I make 90 minutes of calls each month. Which phone company is cheaper?

Ⓓ When is it cheaper to choose 'BLUE' mobile phones?

Personalising learning

Examples of problems which are related to real-life contexts can help increase learners' interest in the subject and help them solve maths problems more confidently. The inclusion of 'real-life' examples motivates learners because abstract concepts become concrete. Humour can help too, such as the example of companies called 'PURPLE' and 'BLUE'.

If possible, for younger secondary school learners it is more meaningful to change problems which involve money calculations from pounds and pence or dollars into the local currency.

Selecting and adapting materials

If teachers decide to write their own maths problems, check that the language is learner-friendly and if not, adapt it by:

- deleting unnecessary words
- using simple sentences rather than complex ones
- simplifying non-subject-specific vocabulary
- avoiding too many modal verbs
- breaking tasks down into smaller steps.

For example:

adapt question Ⓐ

Mobile phone company '*PURPLE*', **has the following charges:** $\$5$ **monthly** rental and $\$0.10$ **per** minute. **If** c is the cost of calls **in a month** and m is the number of minutes of calls in a month then the equation for the cost of the calls is: $c = 0.1m + 5$

① **(reduce length of sentences, delete unnecessary words)**
'*PURPLE*' is a mobile phone company. **It charges**: $\$5$ **each month** for rental and $\$0.10$ **each minute**. The cost of the calls each month is c. The number of minutes each month is m. The equation for the cost of the calls is: $c = 0.1m + 5$

② **(break the task into smaller steps)**

Draw the graph of this (have c on the vertical axis going from 0 to $\$20$ and m on the horizontal axis going from 0 to 120 minutes).

1) Draw the vertical axis from 0 to $\$20$ to show c
2) Draw the horizontal axis from 0 to 120 minutes to show m
3) Now draw the graph of $c = 0.1m + 5$

adapt question Ⓒ

If I make 90 minutes of calls each month, which phone company **should I choose**?

① (*avoid long sentences, avoid modal verbs*)
I make 90 minutes of calls each month. Which phone company **is cheaper**?

adapt question Ⓓ

When **would it be** cheaper to choose *'BLUE'* mobile phones?

① (*simplify language structures*)
When **is it** cheaper to choose *'BLUE'* mobile phones?

Common misconceptions

When one of the coordinate pairs is zero.

e.g. the point $(4,0)$ is often plotted as $(0,4)$

Learners sometimes get the x and y coordinates the wrong way round so watch out for $(3,4)$ being plotted instead of $(4,3)$

Anticipating difficulties

Difficulties learners may have with maths content need to be highlighted. In addition to confusion about plotting points which have one of the coordinates as zero, teachers can help learners remember the correct order of plotting coordinates, by telling them the coordinate order is also in alphabetical order (x then y).

Pointing out errors to learners by making them stand out on the page is also helpful. The use of the exclamation mark draws their attention to the explanation. It is important that page layout is consistent so, as in this example, learners would expect to find all common errors for each topic highlighted by an exclamation mark.

Substituting into equations, BODMAS

e.g. $y = 8 + 2x$ find y when $x = 3$

$y = 8 + 2x$ replace the x by substituting in 3

! $y = 8 + 2 \times 3$ there are two operations to do here: **M**ultiplication and **A**ddition. BOD**MA**S tells us to do the **M**ultiplication before the **A**ddition

$y = 8 + 6$

$y = 14$

!

A common error is to do this sum working from left to right,

e.g. 8 + 2 is 10 then

10 x 3 is 30, which is ***wrong.***

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